

Higher-dimensional Higgs Representations in SGUT models

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Abstract.

Supersymmetric Grand Unified Theories (SGUTs) have achieved some degree of success, already present in the minimal models (with SU(5) or SO(10)). However, there are open problems that suggest the need to incorporate more elaborate constructions, specifically the use of higher-dimensional representations in the Higgs sector. For example, a 45 representation of SU(5) is often included to obtain correct mass relations for the first and second families of d-type quarks and leptons. When one adds these higher-dimensional Higgs representations one must verify the cancellation of anomalies associated to their fermionic partners. One possible choice, free of anomalies, include both 45, $\overline{45}$ representations to cancel anomalies. We review the necessary conditions for the cancellation of anomalies and discuss the different possibilities for supersymmetric SU(5) models. Alternative anomaly-free combinations of Higgs representations, beyond the usual vector-like choice, are identified, and it is shown that their corresponding β functions are not equivalent. Although the unification of gauge couplings is not affected, the introduction of multidimensional representations leads to different scenarios for the perturbative validity of the theory up to the Planck scale. We study the effect on the evolution of the gauge coupling up to the Planck scale due to the different sets of fields and representations that can render an anomaly-free model.

Keywords: Unified theories, Extensions of electroweak Higgs sector, Supersymmetric Models.

PACS: 12.10.Dm, 12.60.Fr, 12.60.Jv, 12.10.Kt

INTRODUCTION

Anomalies in gauge theories. The need to require anomaly cancellation in any gauge theory stems from the fact that their presence destroys the quantum consistency of the theory. It turns out that all one needs to calculate or identify the anomaly is the triangle diagrams. For a given representation of a gauge group G , the anomaly can be written as

$$A(D)d^{abc} \equiv \text{Tr} \left[\left\{ T_a^{D_i}, T_b^{D_i} \right\} T_c^{D_i} \right], \quad (1)$$

where $T_a^{D_i}$ are the generators of the gauge group G in the representation D_i , and d^{abc} denotes the anomaly associated to the fundamental representation [1].

For a representation R that is the direct sum or tensor product of two representations the anomaly coefficient satisfy

$$\begin{aligned} A_R = A(R) &= A(R_1 \oplus R_2) = A(R_1) + A(R_2), \\ A_R = A(R) &= A(R_1 \otimes R_2) = D(R_1)A(R_2) + D(R_2)A(R_1), \end{aligned} \quad (2)$$

respectively, with $D(R_i)$ denoting the dimensions of representation R_i .

Anomaly coefficients. The anomaly coefficients $A(D)$ for most common representations are known in the literature [1]. We have extended these results to include higher-dimensional representation, with some of them shown in Table 1.

TABLE 1. Dimension and anomaly coefficients for higher-dimensional representation of $SU(N)$.

<i>Irrep</i>	$dim(r)$	$A(r)$
	$\frac{N(N-1)(N-2)(N-3)}{24}$	$\frac{(N-4)(N-3)(N-8)}{6}$
	$\frac{N(N+1)(N+2)(N+3)(N+4)}{120}$	$\frac{(N+3)(N+4)(N+5)(N+10)}{6}$
	$\frac{N(N+1)(N+2)(N+3)(N-1)}{120}$	$\frac{(N-2)(N+3)(N+5)^2}{24}$
	$\frac{N^2(N+1)(N+2)(N-1)}{30}$	$\frac{N(N+5)(5N^2-3N-50)}{6}$
	$\frac{N^2(N+1)(N-1)(N-2)}{24}$	$\frac{N(N+5)(5N^2+3N-50)}{24}$
	$\frac{N(N+1)(N+2)(N-1)(N-2)}{24}$	$\frac{N(N-5)(5N^2+3N-50)}{24}$
	$\frac{N(N+1)(N+2)(N-1)(N-2)}{20}$	$\frac{(N^4-17N^2+100)}{4}$
	$\frac{(N-3)(N-2)(N-1)N(N+1)}{30}$	$\frac{(N-5)^2(N-3)(N+2)}{6}$
	$\frac{N(N-1)(N-2)(N-3)(N-4)}{120}$	$\frac{(N-5)(N-4)(N-3)(N-10)}{24}$
	$\frac{N(N+1)(N+2)(N+3)(N+4)(N+5)}{720}$	$\frac{(N+3)(N+4)(N+5)(N+6)(N+12)}{120}$
	$\frac{N(N+1)(N+2)(N+3)(N+4)(N-1)}{144}$	$\frac{(N+3)(N+4)(N+6)(N^2+5N-12)}{24}$
	$\frac{(N-1)N^2(N+1)(N+2)(N+3)}{80}$	$\frac{3}{40}(N-3)N(N+3)(N+4)(N+6)$

TABLE 2. Dimension and anomaly coefficients for different representations of $SU(5)$.

<i>Repr.</i>	<i>Multiplete</i>	$dim(r)$	$A(r)$	$2T(r)$
[5]	(0,0,0,0)	1	0	0
[1]	(1,0,0,0)	5	1	1
[2]	(0,1,0,0)	10	1	3
[1,1]	(2,0,0,0)	15	9	7
<i>Ad</i>		24	0	10
[4,1]	(1,0,0,1)	24	0	10
[1,1,1]	(3,0,0,0)	35	44	28
[2,1]	(1,1,0,0)	40	16	22
[3,1]	(1,0,1,0)	45	6	24
[2,2]	(0,2,0,0)	50	15	35

ANOMALY CANCELLATION.

There are several ways to construct an anomaly free theory:

- The gauge group itself is always free of anomalies. This happens, for instance, for $SO(10)$ but not for $SU(5)$.
- The gauge group is a subgroup of an anomaly free group, and the representations form a complete representation of the anomaly free group. For instance, this hap-

pens in the SU(5) case for the $\mathbf{5} + \overline{\mathbf{10}}$ representations, which together are anomaly free.

- The representations appear in conjugate pairs, i.e., they are vectorlike. This is the most common choice when the Higgs sector of SGUT is extended.

Anomaly cancellation with SU(5) representations. Let us consider an SU(5) SGUT model. There are three copies of $\mathbf{\bar{5}} + \overline{\mathbf{10}}$ representations to accommodate the three families of quarks and leptons. Breaking of the GUT group to the SM is achieved by including a (chiral) Higgs supermultiplet in the adjoint representation ($\mathbf{24}$). The minimal Higgs sector needed to break the SM gauge group can be accommodated with a pair of $\mathbf{5}$ and $\mathbf{\bar{5}}$ representations. Within this minimal model one obtains the mass relations $m_{d_i} = m_{e_i}$, which are predicted by the Higgs sector. One way to solve this problem is to add a $\mathbf{45}$ representation, which couples to the d-type quarks, but not to the u-type, then one obtains the Georgi-Jarlskog factor [3] needed for the correct mass relation. Most models that obtain these relations within an extended Higgs sector, include the $\mathbf{45}$ conjugate representation to cancel anomalies [4]. This is however not the only possibility.

Consider the representations of SU(5) in Table 2. Then, for the $\mathbf{45}$, we can write down the following anomaly-free combinations:

$$\begin{aligned} A(\mathbf{45}) + A(\overline{\mathbf{45}}) &= 0, \\ A(\mathbf{45}) + nA(\mathbf{\bar{5}}) + n'A(\overline{\mathbf{10}}) &= 0, \\ A(\mathbf{45}) + A(\overline{\mathbf{15}}) + 3A(\mathbf{5}) &= 0, \end{aligned}$$

with n and n' integers ≥ 0 and $n + n' = 6$. These are clearly non-equivalent models, with different physical consequences.

GAUGE COUPLING UNIFICATION.

The 1-loop β functions for a general SUSY theory with gauge group G and matter field appearing in chiral supermultiplets are given by $\beta = \sum_R T_R - 3C_A$, where T_R denotes the index for the representation R , and C_A the quadratic Casimir invariant for the adjoint representation. From MSSM-functions, the 1-loop RGEs for the gauge couplings are

$$\frac{d\alpha_i}{dt} = \beta_i \alpha_i^2, \quad \beta_i = \begin{pmatrix} 33/5 \\ 1 \\ -3 \end{pmatrix} + \beta^X. \quad (3)$$

where $\beta^X = \sum_{\Phi} T(\Phi)$ are the contributions of the extensions of the MSSM [5] and the sum is over all SU(5) additional multiplets Φ .

We are interested in evaluating the effect of the different representations in the running from $M_{GUT} \sim 2 \times 10^{16} \text{ GeV}$ up to the Planck scale, and also in finding which representations are perturbatively valid up to the Planck scale. The unified gauge coupling obeys the 1-loop RGE

$$\mu \frac{d\alpha_5^{-1}}{d\mu} = \frac{-\beta}{2\pi} = \frac{3 - \beta^X}{2\pi}. \quad (4)$$

The one loop β functions for some interesting anomaly-free combinations are found to be:

$$\begin{aligned}
\beta^X(45 + \bar{45}) &= 24, \\
\beta^X(45 + 6(\bar{5})) &= 15, \\
\beta^X(45 + 6(\bar{10})) &= 21, \\
\beta^X(45 + \bar{15} + 2(10) + 5) &= 19, \\
\beta^X(50 + \bar{40} + 5) &= 29.
\end{aligned} \tag{5}$$

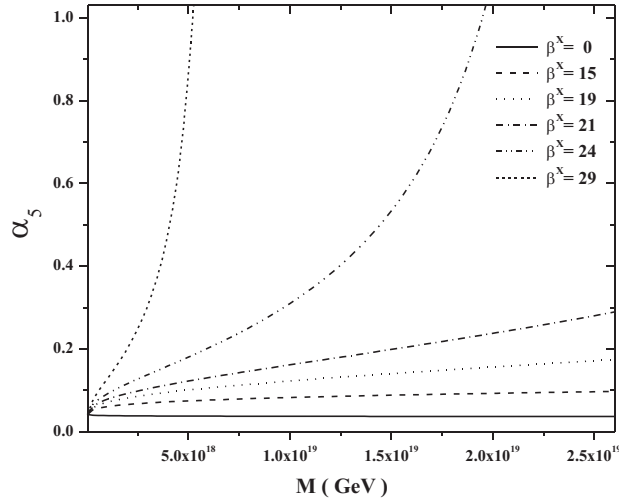


FIGURE 1. Evolution of the gauge coupling for free anomaly combinations listed above, up to the Planck scale .

As it is shown in the figure, the model with $\beta^X = 29$ induces a running of the gauge coupling that blows at the scale $M = 6.61 \times 10^{18}$ GeV, while for $\beta^X = 24$ this happens at $M = 2.63 \times 10^{19}$ GeV. Models with $\beta^X = 15, 19, 21$ are found to evolve safely even up to the Planck scale.

YUKAWA AND GAUGE COUPLINGS.

It is also interesting to consider the RGE effect associated with the Yukawa couplings that involve the Higgs representations. In order to do this we shall consider the 2-loop β functions for the gauge coupling, but will keep only the 1-loop RGE for the Yukawa couplings. Thus, we shall consider the following superpotential for the SU(5) SGUT model [2]. This superpotential involves the Higgs representations $H(5)$, $H(\bar{5})$ and $\Sigma(24)$, and the matter multiplets $\psi(10)$, $\phi(5)$.

$$\begin{aligned}
W = & \frac{f}{3} Tr \Sigma^3 + \frac{1}{2} f V Tr \Sigma^2 + \lambda \bar{H}_\alpha (\Sigma_\beta^\alpha + 3V \delta_\beta^\alpha) H^\beta \\
& + \frac{h^{ij}}{4} \epsilon_{\alpha\beta\gamma\delta\epsilon} \psi_i^{\alpha\beta} \psi_j^{\gamma\delta} H^\epsilon + \sqrt{2} f^{ij} \psi_i^{\alpha\beta} \phi_{j\alpha} \bar{H}_\beta,
\end{aligned} \tag{6}$$

Using the RGEs expressions for a general supersymmetric model [6] we obtain the 1-loop RGE for the Yukawa parameters [2]

$$\begin{aligned}
\mu \frac{d\lambda}{d\mu} &= \frac{1}{(4\pi)^2} \left(-\frac{98}{5} g_5^2 + \frac{53}{10} \lambda^2 + \frac{21}{40} f^2 + 3(h^t)^2 \right) \lambda, \\
\mu \frac{df}{d\mu} &= \frac{1}{(4\pi)^2} \left(-30 g_5^2 + \frac{3}{2} \lambda^2 + \frac{63}{40} f^2 \right) f, \\
\mu \frac{dh^t}{d\mu} &= \frac{1}{(4\pi)^2} \left(-\frac{96}{5} g_5^2 + \frac{12}{5} \lambda^2 + 6(h^t)^2 \right) h^t,
\end{aligned} \tag{7}$$

while the 2-loop RGE for gauge coupling that we obtain is:

$$\mu \frac{dg_5}{d\mu} = \frac{1}{(4\pi)^2} (-3g_5^3) + \frac{1}{(4\pi)^4} \frac{794}{5} g_5^5 - \frac{1}{(4\pi)^4} \left\{ \frac{49}{5} \lambda^2 + \frac{21}{4} f^2 + 12(h^t)^2 \right\} g_5^3. \tag{8}$$

To solve the RGEs we used values of the coefficients λ , h^t and f that are themselves safe at the Planck scale. The parameters used are $M_{GUT} = 1.28 \times 10^{16} \text{ GeV}$, $\alpha(M_{GUT}) = 0.040$, $h^t(M_{GUT}) = 0.6572$, $\lambda(M_{GUT}) = 0.6024$, and $f(M_{GUT}) = 1.7210$.

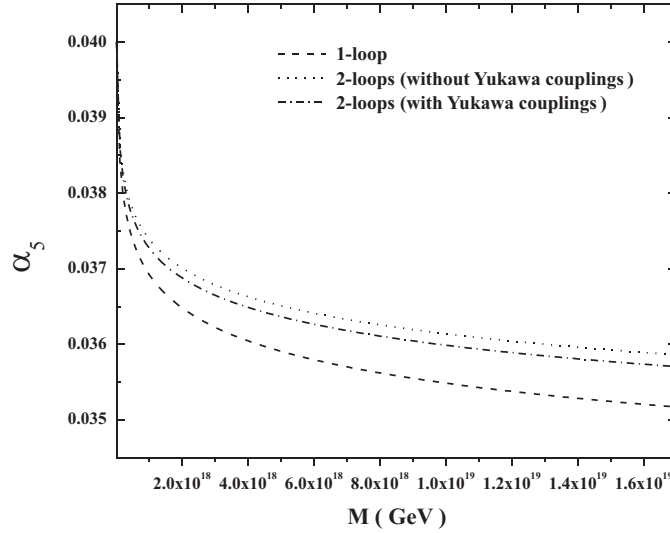


FIGURE 2. Evolution of the unified gauge coupling for three different cases: i) the 1-loop result, ii) the 2-loop result without including Yukawas, and iii) the 2-loop result including the (1-loop) running of the Yukawas.

CONCLUSIONS

We have studied the problem of anomalies in SUSY gauge theories, in order to search for alternatives to the usual vectorlike representations used in extended Higgs sector.

The known results have been extended to include higher-dimensional Higgs representations, which in turn have been applied to discuss anomaly cancellation within the context of realistic GUT models of SU(5) type.

We have succeed in identifying ways to replace the $\overline{45} + 45$ models within SU(5) SGUTs. Then, we have studied the β functions for all the alternatives, and we find that they are not equivalent in terms of their values. These results have important implications for the perturbative validity of the GUT models at scales higher than the unification scale.

We have also considered the RGE effect associated with the Yukawa coupling that involve the additional Higgs representations. We found that there are appreciable differences for the evolution of the gauge coupling when going from the 1 to the 2-loops RGE, but this difference is reduced when one includes the 1-loop Yukawa couplings at the 2-loop level [7, 8].

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